Example 1: A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

Solution:

Let A be the area of rectangular field. Suppose x and y are width and length of rectangle field, then

$$A = xy \qquad \qquad \text{... (i)}$$

Given that the field is of 2400 ft of fencing. SO,

$$2x + y = 2400$$

$$\Rightarrow \quad y = 2400 - 2x \dots \text{(ii)}$$

Thus (i) can be written as

$$A = xy \quad = x(2400 - 2x)$$

$$= 2400x - 2x^2$$

Now,

$$A' = 2400 - 4x$$

For A is maxima, set

$$A' = 0 \quad \Rightarrow \quad 2400 - 4x = 0$$

$$\Rightarrow \quad x = 600$$

Then (ii) gives

$$y = 2400 - 1200 = 1200$$

Also,

$$A'' = -4 < 0$$

This means A is maximum when x = 600 ft and y = 1200 ft.

Thus, length of rectangular field is 1200 ft and width is 600 ft, so that 2400 ft fencing enclosed maximum area.